2023

PHYSICS — HONOURS

Paper : CC-14

(Syllabus : 2019-2020)

[Solid State Physics]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Distinguish between crystalline and amorphous solids with suitable example.

(b) In an orthorhombic crystal a lattice plane cuts intercepts of lengths 3a, –2b and 3c/2 along the three axes. Deduce the Miller indices of the plane.

(c) Consider two ferromagnets : one having a hysteresis curve with broad area and another with a narrow area. Which one can be used as permanent magnet and why?

(d) In a tetrahedral lattice the equal sides are 2.5 Å and the third side is 1.8 Å. Find the lattice spacing between (111) planes.

(e) Taking the origin at the bottom of the conduction band, calculate the crystal momentum for a free electron of energy 0.02 eV. Given, effective mass of electron = 0.2 m_0.

(f) For a superconductor, what do you mean by critical temperature and critical field?

(g) What is dipolar polarisation? How does it depend on temperature?

2. (a) Write down the Laue condition for constructive interference in a crystal and derive the Bragg's law for X-ray diffraction for a simple cubic lattice.

(b) Monochromatic X-rays of wavelength 1.4 Å are incident on a crystal having 1.5 Å as interatomic spacing. Find the maximum order in which the diffraction takes place.

(c) Show that greater the diffraction angle, greater is the accuracy in determining the lattice parameters. (2+3)+3+2

3. (a) Show schematically the acoustic and optical branches due to a linear diatomic lattice in the first Brillouin zone.

(b) Write down the expressions for phase velocity and group velocity of wave motion along a 1-dim lattice. What happens to the group velocity when Ka = $\pm\,\pi$?

(c) Indicate the problem encountered in Einstein's model to explain the behaviour of specific heat of lattice at low temperature. What approximation led Debye to match the observed behaviour at low temperature?

(d) The molar specific heat of a solid at constant volume is 2.77 JK^{-1} mol^{-1} at 36.8 K. Determine the Debye temperature of the solid. 2+(2+1)+(1+2)+2

4. (a) What is the basic assumption of the quantum theory of paramagnetism? For strong magnetic field and at low temperature find out the expression for magnetisation.

(b) The outer electronic configuration of Dy^{3+} ion is $4f^9 6s^0$. Calculate the magnetic susceptibility for a salt containing 1 kg mole of Dy^{3+} at 300 K.

$$\left(\text{Given} : \mu_0 = 4\pi \times 10^{-7}\ \text{NA}^{-2},\ \text{N} = 6.023\times 10^{26},\ \mu_B = 9.27\times 10^{-24}\ \text{JT}^{-1},\ k = 1.38\times 10^{-23}\ \text{JK}^{-1}\right)$$

(2+4)+4

5. (a) The relative permittivity and square of retractive index of a dielectric material are 4.94 and 2.69 respectively. Find the ratio between electronic and ionic polarizabilities of the material.

(b) From the free electron theory find out the temperature dependence of resistivity.

(c) Calculate the Fermi energy at absolute zero of Na-metal (BCC) if its atomic radius is 1.86Å.

$$\left[\hbar = 1.05\times 10^{-34}\ \text{Js},\ m = 9.1\times 10^{-31}\ \text{kg}\right]$$

4+3+3

6. (a) State and explain Bloch theorem.

(b) Show that the effective mass is inversely proportional to the second derivative of the E-K curve.

(c) Discuss, with the help of diagrams, the condition when the effective mass becomes positive, negative and infinity.

(d) The dispersion relation for a 1-d crystal is given by $E(K) = E_0 - \alpha - 2\beta \cos Ka$, where E_0, α, β are constants. Obtain the effective mass at the bottom and top of the band. (2+1)+2+(1+2)+2

7. (a) Derive Hall-coefficient of charge carrier in a metal. Will the Hall-coefficient change sign if one reverses the direction of the applied magnetic field? Explain briefly.

(b) What is isotope effect in superconductivity? Name two experiments from which we can get an idea of existence of energy gap in superconductor.

(c) For lead, the critical field at 0K is 6.39×10^4 A/m and the critical temperature for zero magnetic field is 7.18 K. Find the critical field for lead at 4K. (3+2)+(1+2)+2

Paper : CC-14

(Syllabus : 2018-2019)

[Statistical Mechanics]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) What does a point in a phase space signify? Mention how two phase points differ from one another.

(b) The energy of an oscillator is given $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. From the information that the area of an ellipse with major and minor axes a and b is πab, estimate the volume of phase space between energy E and $E + \Delta E$, drawing a graph and indicating the major and minor axes.

(c) Justify whether we can use equipartition of energy for a Hamiltonian of the form $H = \alpha p^2 + \beta pq + \gamma q^2$.

(d) State two properties of liquid He^4 below the critical point.

(e) The energy levels of a diatomic molecule coming from its rotation motion are given by $\epsilon(j) = j(j+1)\epsilon_0$; $j = 0,1,2,...$ and their degeneracy (multiplicity) is $2j + 1$. Write down the partition function.

(f) Is lamp black a black body? Justify.

(g) The Fermi energy for sodium at $T = 0$K is 3.1 eV. Find its value for aluminium, given that the free electron density in aluminium is approximately eight times that in sodium.

2. Consider a classical ideal gas consisting of N identical and indistinguishable particles of mass m enclosed in a box of volume V.

(a) Calculate the single particle partition function (ξ) and hence write down the partition function (Z) for the system by taking into account the fact that gas molecules are indistinguishable.

(b) Calculate the mean energy of the system.

(c) Calculate Helmholtz free energy of the system and derive Sackur-Tetrode equation.

(2+2)+2+(2+2)

3. (a) "In a microcanonical ensemble the macrostate is defined by fixed number of molecules N, fixed volume V and fixed energy E". Is the statement correct? Justify your answer.

(b) Assuming entropy as a function of number of microstates Ω, show that the additive property of S and multiplicative property of Ω necessarily require $S \propto ln\ \Omega$.

(c) A 1D chain consists of N segments, each of which can be in one of two (non-degenerate) states: horizontal (along the chain) or vertical. Let the length of the segment be a when it is horizontal and 0 when it is vertical. The chain is under fixed tension, so that the energy of each segment be 0 when it is horizontal and ϵ when it is vertical. The temperature of the chain is T. Find the average length of the chain. 2+4+4

4. (a) State Kirchhoff's law of radiation.

(b) Using Planck's expression for the energy density of a black body, obtain Stefan-Boltzmann law.

Given, $$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}.$$

(c) Show that the radiation pressure is thrice the mean energy density of the radiation. 2+4+4

5. (a) Differentiate between a canonical and a grand canonical ensemble.

(b) Using canonical ensemble, establish that $\langle E^2\rangle - \langle E\rangle^2 = K_B T^2 C_V$. Justify that $\sqrt{\langle E^2\rangle - \langle E\rangle^2} \big/ \langle E\rangle$ tends to zero for an ideal gas in the thermodynamic limit.

(c) The magnetic moment of a specimen is given as $\frac{M}{\mu N} = \tanh\left[\frac{1}{K_B T}\left(\mu\mu_0 H + \frac{K_B \theta M}{\mu N}\right)\right]$. If $M << \mu N$ and H is very small, find the temperature dependence of $\chi = \left(\frac{\partial M}{\partial H}\right)_T$. 2+(3+2)+3

6. (a) Show that the Rayleigh-Jeans radiation law is not consistent with the Wien's displacement law.

(b) At a given temperature, $\lambda_{max} = 6500$ Å for a black body cavity. What will λ_{max} be if the temperature of the cavity walls is increased, so that the rate of emission of spectral radiation is doubled?

(c) Show that in canonical ensemble, the average energy of the system and the pressure in terms of the partition function (Z) are given by

(i) $\bar{E} = -\frac{\partial \ln Z}{\partial s}$

(ii) $\bar{P} = \frac{1}{s}\frac{\partial \ln Z}{\partial V}$. 3+3+(2+2)

7. (a) There are 3 particles in 4 quantum states. Distribute the particles according to BE and FD statistics.

(b) Name the statistics to be followed by each of the following particles with proper reasoning : neutron, neutrino, Hydrogen atom, π-meson.

(c) Consider two indistinguishable particles which may exist in two different states a and b. State with reason whether the wave function $\psi = \frac{1}{\sqrt{2}}\left[\psi_a(1)\psi_b(2) - \psi_a(2)\psi_b(1)\right]$ represents a Boson or a Fermion.

(2+2)+4+2
